Comments on Two-Phase Measurements Using a Resistive Probe

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A timely contribution to two-phase measurement techniques is described by Bankoff and Neal (*1*). Their resistive-probe technique for measuring local void fractions should prove valuable to those who desire to know more about local properties in two-phase flow. In reference *1*, the authors show expressions for the volumetric fraction of the gas, the median chord of a bubble pierced by the probe, and size distributions of gas and liquid. Their development treats the expressions separately. For any bubbles, the median chord is independent of motion, since it is obtained from the one-half total volume of a bubble. The average chord represents the ratio of total volume to area normal to the flow, and hence depends on fluid motion. The dependency links it to volumetric fraction and size distributions.

Using the concept of average bubble chord, these comments present the development of relationships between local time-averaged and space-averaged properties and illustrate the difficulties of interpreting in terms of bubble size the probe time traces caused by spherical gas bubbles. Possibilities of future work in local measurements are discussed.

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Bubble Radius Distribution Functions from Resistivity Probe Measurements

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Anderson (*1*) derives a formula relating the bubble radius probability density function (*2*) to the contact-time density function on the assumption that the bubble-rise velocity is independent of position; that is, $U = U(\zeta)$. It is probably more realistic in most systems to assume that $U = U(\zeta, s)$. Here a formula for this situation is derived by a method somewhat different than the one employed by Anderson.

Assume that the probe is positioned at a dimensionless radial position, s_o. Also, with respect to polar coordinates fixed at the center of the bubble, the z axis pointing upward and the x-axis pointing toward the center of the pipe, the position of the probe tip at the instant of contact with the bubble is (ζ, θ, ϕ). The triangle cosine law then requires that

$$s_o^2 = \zeta^2 \sin^2\theta + s^2 - 2s\zeta \sin\theta \cos\phi \tag{1}$$

Now the joint probability density function of the simultaneous event consisting of a particular combination of the random variables ζ, θ, and ϕ is given by

$$f(\zeta, \theta, \phi) = [b(\zeta)] [2\cos\theta \sin\theta] \left[\frac{1}{2\pi}\right] \tag{2}$$

where it is assumed that the three events are unrelated, so that their joint probability is the product of the individual probability densities; that any element of the projected bubble frontal area has the same probability of being struck as any other element; and that the azimuthal angle probability density is a constant.

Consider first the case where the bubble-rise velocity depends only on bubble radius; that is, $U = U(\zeta)$. It is convenient to transform the joint probability density function to a new set of random variables (ζ, τ, ϕ) by evaluating the Jacobian $J_1 = \dfrac{\partial(\zeta, \theta, \phi)}{\partial(\zeta, \tau, \phi)}$ where the contact time is given by

$$\tau = \frac{2R\zeta\cos\theta}{U} \tag{3}$$

assuming that the bubble is not deflected or slowed down by the probe. In this way it is found that

$$J_1 = \frac{\partial\theta}{\partial\tau} = -\frac{U}{2R\zeta\sin\theta} \tag{4}$$

which leads to

$$f(\zeta, \tau, \phi) = \frac{b(\zeta)\tau}{\pi}\left(\frac{U}{2\zeta R}\right)^2 \tag{5}$$

The probability density function, $p(\tau)$, for bubble contact times is a marginal distribution function (*3*) obtained by integration over two dimensions:

$$p(\tau) = 2\tau\int_{\zeta_{\min}}^{\zeta_{\max}} \left(\frac{U}{2R\zeta}\right)^2 b(\zeta)\,d\zeta \tag{6}$$

This is the same as Anderson's Equation (2). However, the bubble-rise

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INFORMATION RETRIEVAL

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Permeation of gases through polyethylene films at elevated pressures, Li, N. N., and E. J. Henley, **A.I.Ch.E. Journal, 10,** No. 5, p. 666 (September, 1964).

Key Words: Permeation-8, Methane-1, 2, Ethane-1, 2, Propane-1, 2, Butane-1, 2, Nitrous Oxide-1, 2, Carbon Dioxide-1, 2, Dichlorodifluoromethane-1, 2, Polyethylene-5, Pressure-6, Temperature-6, Henry's Law-9, Fick's Law-9, Boiling Point-9, Solubility Parameter-9, Polymer-10, Diffusion-9, Sorption-9, Membrane Thickness-6, Arrhenius Equation-9, Crystallite-5, Permeability-7, Interaction-6, Eyring Theory-9, Plasticizing-9, Condensability-9, Organic-, Inorganic-, Polar-, Nonpolar-, Saturated-, Unsaturated-.

Abstract: Permeation of methane, ethane, ethylene, propane, butane, nitrous oxide, carbon dioxide, and dichlorodifluoromethane through polyethylene films was investigated.

Both Henry's law and Fick's law are obeyed for carbon dioxide permeation through polyethylene at pressures up to 11 atm. and over a temperature range from 14° to 40°C. For other gases permeability is pressure dependent and is described by $P = P_0 \exp (Ap)$. Based on this expression a modified diffusion law is proposed.

P_0 for nonpolar, organic gases is correlated by their boiling points. Constant A is correlated by the Hildebrand solubility parameters of the gas and polymer.

The additivity of individual phase resistances in mass transfer operations, King, C. Judson, **A.I.Ch.E. Journal, 10,** No. 5, p. 671 (September, 1964).

Key Words: Mass Transfer-8, Addition of Resistances-8, Surface Lifetime Distribution-6, Penetration Theory-6, 9, Two-Film Model-9, Mass Transfer Coefficient (Overall)-7, Mass Transfer Coefficients (Local)-6, Packed Tower-10, Plate Tower-10, Design-9.

Abstract: Necessary conditions are outlined for the simple addition of resistances equation to be valid in a mass transfer process. Several simple models are examined to determine the extent of deviation from this equation. A major effect can come from a large local variation in Hk_G'/k_L', which can result from a wide distribution of liquid and/or vapor surface lifetimes. This concept is capable of resolving the observed discrepancies between vaporization and absorption data in packed and plate towers and predicts a continuous change in the effective area for mass transfer as the degree of gas or liquid-phase control changes.

Statistical surface thermodynamics of simple liquid mixtures, Eckert, C. A., and J. M. Prausnitz, **A.I.Ch.E. Journal, 10,** No. 5, p. 677 (September, 1964).

Key Words: A. Surface Tension-8, 2, Thermodynamics-10, Statistical Mechanics-10, Partition Function-10, Interface-9, Mixtures-5, Cell Model-10, Configurational Energy Density-1, Surface Area Fraction-1, Surface Composition-2, Adsorption-2. B. Surface Tension-7, Surface Composition-7, Mixtures-5, Cryogenic-9, Argon-5, Nitrogen-5, Oxygen-5, Methane-5, Carbon Monoxide-5, Benzene-5, Neopentane-5, Cyclohexane-5, Carbon Tetrachloride-5.

Abstract: The surface properties of nonpolar liquid mixtures are derived from the application of the grand partition function to a cell model of the vapor-liquid interface. The surface tension and the surface composition are expressed in terms of the activity coefficients in the bulk liquid and pure-component properties. The results are in excellent agreement with existing data for the surface tension of mixtures. This treatment provides a sound theoretical basis for the prediction of surface properties of other liquid systems and is especially applicable to cryogenic mixtures.

Boiling burnout heat flux measurements in a non-Newtonian suspension, Eissenberg, D. M., **A.I.Ch.E. Journal, 10,** No. 5, p. 684 (September, 1964).

Key Words: Boiling-8, Subcooling-6, Thoria-10, Non-Newtonian-8, Burnout Flux-8, 7, Suspension-10, Slurry-10, Pool-10, Yield Stress-6, Heat Transfer-7, Convection-10, Pressure-6.

Abstract: Boiling burnout heat fluxes of thin stainless steel strips immersed in non-Newtonian aqueous suspensions of thoria were measured at saturation temperature and with subcooling for atmospheric pressure pool boiling and high pressure forced convection. A decrease in the burnout heat flux, especially with subcooling, was attributed to the suspension yield stress.

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VOLUMETRIC VOID FRACTION FOR IRREGULAR BUBBLES

In reference *1*, the local void fraction is defined as the probability that gas exists at a point in the channel. From this definition, for a long enough measurement interval τ_m, the fraction of time that gas contacts the probe is also the void fraction. The time average of gas contacting the probe may be expressed in terms of bubble sizes and velocities to transform into space averages of gas and liquid volumes. In the following development, the actual behavior of irregular masses of bubbles is simplified, and the effects of pipe walls on the distributions and velocities of bubbles is neglected. If δ is the bubble chord seen by the probe and U is the velocity of the bubble, the time gas contacts the probe is given by

$$\theta = \delta / U \tag{1}$$

For a large number of bubbles of approximately the same shape, size, and velocity, the average bubble transit time is

$$\bar{\theta} = \bar{\delta} / U \tag{2}$$

The average bubble chord is found easily if the bubble has equal probability of striking the probe anywhere over its area:

$$\bar{\delta} = \left(\int_A \delta \, dA \right) \Big/ \left(\int_A dA \right) = V/A \tag{3}$$

With the average bubble chord defined, the space dependence from time traces may be obtained. For irregular shapes of bubbles, discrete populations may be assumed and an expression for void fraction derived (Figure 1). The same procedure is applied to a continuous distribution of spherical bubbles.

VOLUMETRIC VOID FRACTION AND SIZE DISTRIBUTIONS OF SPHERICAL BUBBLES

The resistive probe may be used for more than determining the gas and liquid volumetric fractions. Analysis of pulses from the probe yields information about bubble sizes and bubble distribution in the liquid. This information would be useful in future studies on the basic structure of two-phase flow. The following development treats spherical bubbles to show some relationships that would be useful in these studies. If the velocities of the spherical bubbles were determined, their sizes could be found. The analysis of spheres should provide an upper bound on the difficulty of determining size distributions, since the maximum surface curvature of a sphere will cause a corresponding maximum variation of chord length about the average.

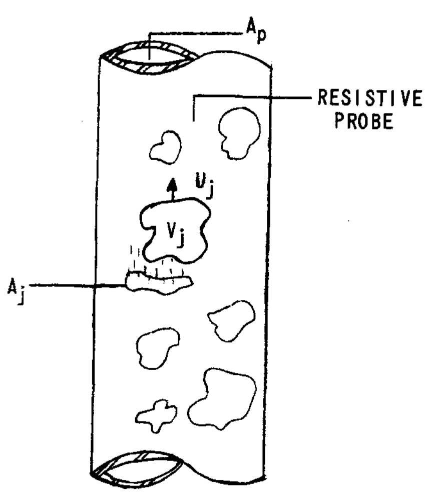

Fig. 1. Irregularly shaped bubbles in a pipe.

An idealized analysis indicates the effects of the bubble size distribution on the gas contact time and on the volumetric void fraction. For the purpose of illustration, the actual behavior of bubbles moving in a liquid is approximated by an infinite-medium solution. Influence of pipe wall on all aspects of bubble size and motion is neglected, bubbles are uniformly distributed across the area perpendicular to flow, and all bubbles of the same radius move at the same velocity. For these conditions, the average bubble chord pierced by the probe has a length of $(4/3)r$. The variance or standard deviation of the chord about the average is $(\sqrt{2}/3)r$, or roughly half the radius. This variance is large enough to create difficulties in obtaining bubble size directly from the traces.

The bubble size distribution function, $B(\zeta)$, discussed by Bankoff and Neal [their Equation (2.2)] is reiterated here:

$$B(\zeta) = \int_0^{\zeta} b(Z)\, dZ \qquad (4)$$

where $Z = r/R$ and $B(\zeta)$ is the probability that the nondimensionalized radius of a bubble striking the probe is less than ζ. The density function, $b(\zeta)$, is the derivative of $B(\zeta)$ with respect to ζ.

To obtain a simple expression for void fraction in terms of the bubble size distribution, a few assumptions will be made. The assumptions are not crucial to the analysis but illustrate the procedure of obtaining void fraction for a simple case. For an infinite flow field, velocity and size of the bubble are independent of the bubble position relative to the probe. If the bubbles are noninteracting, bubble velocity should be a function of radius only (Figure 2). The average bubble transit time for a bubble of radius r is then

$$\bar{\theta}(r) = \frac{\bar{\delta}\,(r)}{U\,(r)} \qquad (5)$$

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glects consideration of the movement of the solid in the bed which is important in the chemical reactions in many cases.

The reader will find the material and presentation interesting and provocative, but the state of the art appears to be more appropriate for journal articles and symposia than for a book.

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Process Control, Peter Harriott, McGraw-Hill, New York (1964). 374 pp. $13.50.

This book has arrived on the scene just in time to fill a vital need for the professor of chemical process dynamics and control and for his student. It provides an intermediate level text which combines, in an integrated fashion, both automatic control theory and the dynamic response of chemical plant apparatus and its associated control system.

Previously one could choose only from the elementary texts in the field, such as those of N. H. Ceaglske, G. D. Shilling, or T. J. Williams and V. A. Lauher, or from the quite advanced and specialized texts such as Campbell's *Process Dynamics*. Many thus chose one of the many excellent texts of the mechanical or electrical engineering fields at the sacrifice of applications and examples from the chemical field. Therefore Professor Harriott's text should find a welcome reception at our universities and colleges.

This book further justifies its choice as a textbook by a natural breakdown into three progressively more comprehensive and more difficult areas of discussion; by a wide choice of problems at the end of each chapter; by the use of excellent illustrative examples and figures, and by a quite complete author and subject index and table of nomenclature and symbols.

Chapters 1 through 7 of the book can be considered as a review of the basic concepts of automatic control with attention focused on the mathematics of automatic control and on the use and the dynamic response of typical automatic control equipment. While similar to the treatment given in any good text on automatic control, regardless of the field of engineering involved, this section does correlate the theory quite well to the field of chemical engineering through the use of examples taken from the chemical process area.

A second section comprised of Chapters 8 through 13, less 11, and Chapter 16 presents a comprehensive view of several of the specialized areas of the chemical process control field such as control valves and their use, level control, flow control, pH control, theory and use of complex control schemes such as cascade control and feedforward control, and methods of obtaining optimum controller settings.

The third section, Chapters 11, 14, and 15, gives excellent reviews and discussions of the three major areas of chemical process dynamics and control investigation today—heat exchangers, distillation columns, and chemical reactors.

Despite its overall excellence, the book does have some drawbacks which perhaps should be called to the reader's attention for his evaluation of their importance. Foremost of these concerns the problems following each chapter. Many of them might be difficult for the student at the level of knowledge he will presumably have when he encounters them. Several others call for the use of data from the literature or the reference to other independent textbooks, some of which are relatively old. Second of this reviewer's concerns relates to the author's choice of subject matter. It would be a major aid to the student if the subject of temperature control in general could have been treated to the extent of level control, flow control, etc. This is particularly important since such systems tend to have an entirely different level of time constants, etc. The subject of heat exchangers is well covered but is only a part of the temperature control field. Likewise, the subject of analytical instrumentation application and response, which is becoming so vital to chemical process control, is effectively limited to the discussion of pH control in Chapter 16. Perhaps both of these important subjects will be greatly expanded if and when a second edition is prepared.

The textbook *Process Control* is thus one which is highly recommended for any two-semester senior level or beginning graduate student course in chemical process dynamics and control.

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Advances in Heat Transfer—Vol. 1, Edited by Thomas F. Irvine, Jr., and James P. Hartnett, Academic Press, New York (1964). 459 pages. $18.00.

The publishing industry provides a useful service to the scientific community and earns a significant income by marketing collections of review articles which are variously entitled "Progress In . . .," "Annual Reviews of . . .," "Modern Developments In

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INFORMATION RETRIEVAL

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The dynamics of flow forced distributed parameter heat exchangers, Stermole, F. J., and M. A. Larson, **A.I.Ch.E. Journal, 10,** No. 5, p. 688 (September, 1964).

Key Words: Concentric Double Pipe Heat Exchanger Dynamics-10, Countercurrent Heat Exchanger Dynamics-10, Double Pipe Heat Exchanger Dynamics-10, Dynamic Response of Heat Exchangers-8, Flow Forced Heat Exchangers-1, Frequency Response of Heat Exchangers-8, Heat Exchanger Dynamics-10, Heat Transfer Dynamics-8, Process Dynamics-8, Process Dynamics-8, Thermal Process Dynamics-1, Transfer Functions of Heat Exchangers-1, Transient Response of Heat Exchangers-8, Unit Operations Dynamics-8.

Abstract: Dynamic response of a concentric double pipe countercurrent heat exchanger subjected to flow rate changes was studied experimentally and theoretically. Both large and small flow perturbations about a mean flow rate gave the same frequency response curves indicating linear system behavior in the flow range investigated. Wall capacitance and heat transfer coefficient variation were found to have little theoretical effect on normalized frequency response results, but heat transfer coefficient variation contributed significantly to transient response results. Partial differential equation transfer function models gave good agreement with experimental frequency response results but only agreed with experimental transient results for small flow upsets.

Viscosity correlation for light hydrocarbon systems, Lee, A. L., K. E. Starling, J. P. Dolan, and R. T. Ellington, **A.I.Ch.E. Journal, 10,** No. 5, p. 694 (September, 1964).

Key Words: Viscosity-7, Generalized-8, Correlations-8, Behavior-9, Hydrocarbons-9, Density-6, Temperature-6, Pressure-6, Least Squares-10, Methane-9, Ethane-9, Propane-9, *n*-Butane-9, Mixtures-9.

Abstract: A generalized equation for predicting the viscosity behavior of light hydrocarbons and their mixtures has been developed. The behavior of methane, ethane, propane, *n*-butane, methane-propane, and methane-*n*-butane mixtures can be predicted with a calculated standard deviation of the data of less than 2%.

An analysis of the equilibrium stage separations problem—formulation and convergence, Friday, John R., and Buford D. Smith, **A.I.Ch.E. Journal, 10,** No. 5, p. 698 (September, 1964).

Key Words: Convergence-2, Distillation-8, Extraction-8, Absorption-8, Stripping-8, Washing-8, Flash-8, Equilibrium-8, Separations-8, Computer-10, Error-2, Stability-2.

Abstract: The decisions involved in formulation of a solution method for equilibrium stage separations are analyzed. A solution method for absorption and extraction problems which proved successful is formulated. The reasons for failure of distillation methods to handle absorption or stripping problems are discussed. The criterion for asymtotic stability of successive substitution procedures is applied to example problems. Error build up is discussed and one means of avoiding it is presented.

Surface combustion of hydrogen: part I. On platinum-coated alumina, Gidaspow, Dimitri, and Rex T. Ellington, **A.I.Ch.E. Journal, 10,** No. 5, p. 707 (September, 1964).

Key Words: Surface Combustion-8, Catalysis-8, Oxidation-8, Hydrogen-1, Air-1, Reactor (Tubular)-10, Platinum-4, Temperature-6, Concentration-6, Flow Rate-6, Roughness-6, Reaction Rate-7, Heat Transfer Coefficient-7, Turbulent.

Abstract: The rate of surface combustion of hydrogen in excess air at atmospheric pressure on platinized vitreous alumina is proportional to the hydrogen concentration. It rapidly increases with surface temperature up to about 450°F. and then slowly decreases with temperature up to 1,600°F. Heat transfer analysis has shown that the reaction generally takes place on the surface. Differences in reaction rates and heat transfer coefficients along the tube, excluding the end section, were ascribed to variable surface roughness.

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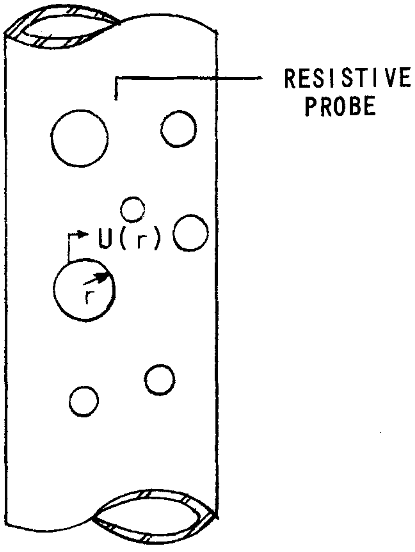

Fig. 2. Spherical bubbles in a pipe.

Suppose that in measurement time, τ_m, N bubbles strike the probe. For large N, the proportion of bubbles between r and $r + dr$ is the bubble density distribution:

$$b(\zeta) \simeq \left(\frac{dn}{dr}\right) \quad n \qquad (6)$$

For a cylindrical element of radius R that contains all the bubbles striking the probe, the probability of the existence of a bubble is uniform over the flow cross section of the element. The probability that a bubble of radius r within the cylinder strikes the probe is the ratio of projected bubble area to cylinder area:

$$P_1 = \frac{\pi r^2}{\pi R^2} \qquad (7)$$

The product of bubble strike probability, P_1, average bubble contact time, $\overline{\theta}(r)$, and the differential number of bubbles of radius r, $\frac{dn}{dr}$, is the contribution to gas contact time by the bubbles between radii r and $r + dr$. Integrating over r yields the total gas contact time

$$\tau_g = \int_o^{r_{\max}} P_1 \, \overline{\theta}(r) \frac{dn(r)}{dr} dr \qquad (8)$$

The void fraction α is τ_g/τ_m, or

$$\alpha = \frac{1}{\tau_m} \int_o^{r_{\max}} \frac{r^2}{R^2} \frac{\overline{\delta}(r)}{U(r)} \frac{dn(r)}{dr} dr \qquad (9)$$

where $r_{\max}$ is the maximum bubble radius. The rate of bubbles striking the probe is

$$q = \frac{n}{\tau_m} \qquad (10)$$

Nondimensionalizing the equation in terms of the characteristic radius $\zeta = r/R_A$, and substituting Equations (6) and (10) into (9), the void fraction may be written as

$$\alpha = q \int_{o}^{\zeta_{\max}} \{[\bar{\delta}(\zeta)\, b(\zeta)]/U(\zeta)\} d\zeta \tag{11}$$

It should be noted that $b(\zeta)$ is a measurable quantity, but that it represents only the population of bubbles striking the probe. There are more small bubbles existing in the pipe than are indicated by $b(\zeta)$, since the larger bubbles have a better chance of striking the probe. If a distribution function for all bubbles crossing the plane of the probe were designated $H(\zeta)$, then

$$H(\zeta) = \int_{o}^{\zeta} h(Z)\, dZ \tag{12}$$

where $h(\zeta)$ is the probability that the bubble radius is less than ζ. The quantity $h(\zeta)$ could be determined by photographic studies. When the assumption of uniform distribution of bubbles over pipe area is used, the expression for void fraction is

$$\alpha = Q \int_{o}^{\zeta_{\max}} \{\bar{\delta}(\zeta)\, \zeta^2\, h(\zeta)/U(\zeta)\} d\zeta \tag{13}$$

The relationship between density functions for bubbles passing the plane of the probe is

$$q\, b(\zeta) = Q\, \zeta^2 h(\zeta) \tag{14}$$

The point of the discussion of h is that bubble size distributions from probe contact and from photographs differ by a bubble area weighting factor.

BUBBLE TRANSIT TIMES FROM THE DENSITY FUNCTIONS

The output signal from the probe consists of pulses whose time durations are bubble transit times. If bubble shapes and velocities are known, these transit times may be used to determine the bubble size distribution. Reference *1* presents Equation (3.5) which relates transit time to bubble diameter by

$$D_B = C_1\, \theta_B\, U_B \tag{15}$$

The constant C_1 depends on bubble size and probe distance from the pipe wall. If the probe-to-wall distance is less than the bubble radius, the average chord pierced by the probe is shorter than the length defined in Equation (3), causing C_1 to be greater than for smaller bubbles. For spherical bubbles not influenced by the pipe wall, C_1 becomes 3/2. The following discussion will consider only the case of C_1 not influenced by the pipe wall.

The simplest case is that of spherical bubbles possessing the same radius and velocity. The time-duration distribution, $P(\theta)$, is given by

$$P(\theta) = \int_{\tau=0}^{\theta} p(\tau)\, d\tau \tag{16}$$

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. . .," "Advances In . . .," etc. It is the expressed intent of the authors of this latest addition to the "Advances In . . .," to bridge the gap between the regularly scheduled journals and the university-level textbooks and enable a nonspecialist reader to make engineering use of highly condensed journal articles.

The primary function of the editors is to prevail upon authors of competence to undertake the effort to produce the review and deliver it by a deadline. Absence of further editorial action as indicated by differences in style, nomenclature, and literature citations is generally the rule and is also true of this work.

In their primary responsibility the editors have chosen well in selecting topics and authors but show a heavy bias toward academic contributors. Authors from Russia and Italy give the book an international flavor, and the bibliographies provide a lead into the continental literature for those who tend to consider only English language publications.

The six contributions are summarized briefly below.

"The Interaction of Thermal Radiation with Conduction and Convection Heat Transfer," by R. D. Cess, is an exposition of the effects of the interactions rather than a comprehensive treatment of the entire problem. By the use of simple models, estimates of the magnitudes of effects are illustrated. The error of assuming superposition for grey bodies is emphasized. Only literature of the United States is cited and comparison made to only one experiment.

"Application of Integral Methods to Transient Nonlinear Heat Transfer," by Theodore R. Goodman, is a *tour de force* in approximate methods of solving partial differential equations. The emphasis is on applied mathematics and various approximations are compared by numerical examples. The techniques described are equally applicable to problems other than heat transfer, and this review is highly recommended to the mathematically inclined.

"Heat and Mass Transfer in Capillary-Porous Bodies," by A. V. Luikov, is probably the weakest contribution in the volume. The English is awkward, and the inclusion of detailed experimental description and raw tabulated data detracts from the review nature of the article. Inclusion of theoretical information readily available in reference *19* is superfluous, and neglect of important work in western literature is inexcusable. The editors appear briefly in a footnote on page 137, and this reviewer wished their hand had been heavier and the article more representative of the well-deserved reputation of Professor Luikov.

"Boiling," by George Leppert and C. C. Pitts, could probably be expanded into a book. It represents the continuing work of Professor Leppert since his own early involvement in the Atomic Energy Commission's water heat transfer program and is a welcome updating of J. W. Westwater's classic review. The whole gamut of surface phenomena, bubble dynamics, stability of interfaces, and effect of gravity is covered. Many excellent photographs are used, and extensive reference is made to report literature. Because report literature is important, it is difficult for someone just entering such a field to have access to the source of the reports. This makes their abstraction and correct citation more urgent.

"The Influence of Electric and Magnetic Fields on Heat Transfer to Electrically Conducting Fluids," by Mary E. Romig, is a sleeper. The author will probably become better known for this outstanding review of a new and rapidly expanding area. This scholarly study cuts across the boundaries of hypersonic flow as well as natural and forced convection and includes a concise summary of the principles of MHD. Recent publications in the area of this topic makes one wish the cutoff date in the bibliography had not been early 1962 but early 1963, since the book was published in 1964. Inclusion of the effect of strong electric fields on nonconducting liquids would also have been desirable.

"Fluid Mechanics and Heat Transfer of Two-Phase Annular-Dispersed Flow," by Mario Silvestri, is the longest contribution (92 pages) and the only one to add a supplementary list of references. The text is divided almost equally between heat transfer and fluid mechanics. Professor Silvestri liberally illustrates his work with experimental details and results from his own laboratory. All units used in formulas or illustrations are metric regardless of the origin of the work, although British units are often quoted in parallel. The literature survey is diverse and covers European, Russian, and English langauge publications and reports. The only significant flaw in this review is the lack of care shown in the spelling of names and proper citation of literature. This lack of care is illustrated by the double inclusion of the paper of Owens on page 445.

The carelessness in spelling and transliteration of Russian names is also evident in the rather complete author index where duplicate listings of au-

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INFORMATION RETRIEVAL

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Surface combustion of hydrogen: part II. On oxidized nickel, Gidaspow, Dimitri, and Rex T. Ellington, **A.I.Ch.E. Journal, 10,** No. 5, p. 714 (September, 1964).

Key Words: Surface Combustion-8, Catalysis-8, Oxidation-8, Adsorption Mechanism-8, Hydrogen-1, Air-1, Reactor (Tubular)-10, Nickel Oxide-4, Nickel-4, Temperature-6, Concentration-6, Reaction Rate-7.

Abstract: The rate law for surface combustion of hydrogen in excess air on oxidized nickel at an absolute pressure of 49 in. Hg and at a surface temperature of 1,600°F. can be represented as 1.8 order at concentrations from 0.2 to 1.5 mole % hydrogen and by 1.35 order from 1.5 to 4.0 mole % hydrogen. The rate of reaction increases with surface temperature in the range of 1,300° to 2,000°F. An adsorption mechanism is proposed. The rate controlling step may be the reaction of two adsorbed molecules of hydrogen with one molecule of oxygen in the gas phase.

Kinetics of particle growth in the fluidized bed calcination process, Grimmett, Earl S., **A.I.Ch.E. Journal, 10,** No. 5, p. 717 (September, 1964).

Key Words: Particles-2, 7, 8, 9, 10, Growth-7, 9, Fluidized Bed-8, 9, 10, Calcination-8, 9, 10, Radioactive Wastes-1, 2, 8, Size-6, Theory-8, 9, Correlations-8, 10, Comparison-8, 9, Measuring-8, 9, 10, Aluminum Nitrate-1, Nitric Acid-1, Aluminum Oxide-2, Nitrogen Oxide-2, Heating-1, 10, Rates-1, 2, 6, 8, 9, Fluidizing Air-5, 2, Tracer-4, 8, 9, 10, Boric Acid-4, Sodium Nitrate-1, 3, Mercuric Nitrate-1, 3.

Abstract: The rate of particle growth in the fluidized bed calcination process was studied with the aid of several techniques which are described and evaluated. These studies show that the growth rate for small particles in this process is less than that for large particles. A particle growth rate equation is presented which is based on fundamental relationships between calciner operating variables and includes a factor which corrects for the effect of particle size.

Velocity profiles of thoria suspensions in turbulent pipe flow, Eissenberg, D. M., and D. C. Bogue, **A.I.Ch.E. Journal, 10,** No. 5, p. 723 (September, (1964).

Key Words: Velocity Profile-10, Non-Newtonian-9, Turbulent Flow-9, Thoria-1, Suspension-9, Slurry-9, Pipe Flow-8, Water-5, Correlations-9, Friction Factors-10, Fluid Flow-8.

Abstract: Friction factors and velocity profiles were obtained for a dilute and a concentrated aqueous thoria suspension and compared with the Newtonian correlations and with previous non-Newtonian data. A relationship was found between the decrease of the friction factor and the change in the velocity distribution toward a more laminar profile. A generalized method of deriving the friction factor-Reynolds number plot from the velocity profile based on the von-Karman relationship for Newtonian fluids is derived and compared with the data.

An ionic penetration theory for mass transfer with chemical reaction, Brian, P. L. T., R. F. Baddour, and D. C. Matiatos, **A.I.Ch.E. Journal, 10,** No. 5, p. 727 (September, 1964).

Key Words: Gas-5, Liquid-5, Reaction Rate-6, Diffusivity-6, Mass Transfer-8, Diffusion-8, Reaction Kinetics-8, Ion Diffusion-8, Mathematics-10, Computations-10, Finite-Differences-10, Penetration Theory-10.

Abstract: A theoretical treatment is given for the rate of mass transfer of a molecular species into an aqueous solution containing ions, one of which reacts chemically with the transferring component. Using the penetration theory model, it is shown that ionic diffusion effects upon the rate of mass transfer are appreciable. It is also indicated how satisfactory approximations to the penetration theory results can be obtained by means of the film theory. The computational effort required for this procedure is much smaller than that needed for solving the penetration theory equations themselves.

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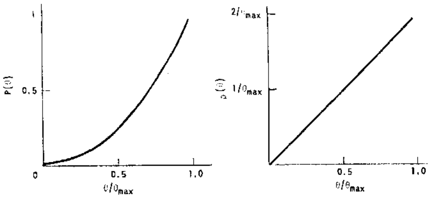

Fig. 3. Distribution and density functions for bubbles of radius, r.

Where $P(\theta)$ is the probability, a pulse has duration shorter than θ. The bubble area corresponding to the time interval τ to $\tau + d\tau$ is

$$2\pi\rho d\rho = -\frac{U^2}{4}(2\pi\tau d\tau) \quad (17)$$

The time-duration distribution becomes

$$P(\theta) = \int_{(\rho/r)_{\min}}^{1} 2(\rho/r)\, d(\rho/r)$$

$$= \int_0^\theta [U^2/(4r^2)]\,(2\tau d\tau) \quad (18)$$

The distribution for this case (Figure 3) is identical to the distribution of bubble chords.

The determination of a bubble size distribution for spherical bubbles from time traces involves conditional probabilities (2). The relationship between bubble size and the time traces may be determined with less use of conditional probability by a procedure similar to that of the preceding example. That is, given the bubble size distribution, one may find the resulting time-duration distribution. For a density function, $b(\zeta)$, the fraction of bubbles from the population of radius r contributing to $d\tau$ is

$$[2b(\zeta)\,\rho d\rho]/(r^2) =$$

$$-\,[2U^2\, b(\zeta)\,\tau d\tau]/(4r^2) \quad (19)$$

The probability that τ is less than θ is found by integrating first over r, then over τ. In terms of ζ

$$P(\theta) = \int_{\tau=0}^{\theta} \left\{ \int_{\zeta_{\min}}^{\zeta_{\max}} [\mu^2(\zeta)\, b(\zeta)\, d\zeta]/(4\zeta^2) \right\} 2\tau\, d\tau \quad (20)$$

The lower limit of the inner integral may be expressed in terms of τ, since $\zeta_{\min}$ is given by

$$\zeta_{\min} = [\tau U\,(r_{\min})]/(2R) \quad (21)$$

The time-duration density function, $p(\theta)$, is found by differentiating $P(\theta)$ with respect to θ. To show the relationship of the time-duration density function to size density function, two examples are given; $b(\zeta)$ equals a constant (Figure 4) and $b(\zeta)$ is proportional to ζ^2 (Figure 5).

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INFORMATION RETRIEVAL

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Hydrates at high pressures: Part II. Application of statistical mechanics to the study of the hydrates of methane, argon, and nitrogen, Saito, Shozaburo, Donald R. Marshall, and Riki Kobayashi, **A.I.Ch.E. Journal, 10,** No. 5, p. 734 (September, 1964).

Key Words: A. Hydration-8, Hydrocarbon Hydrate-1, Argon Hydrate-1, Methane Hydrate-1, Nitrogen Hydrate-1, High Pressure-5, Physical Properties-6. B. Statistical Mechanics-8, Thermodynamics-8, Enthalpy-9, Heat of Formation-9, Hydrate Number-9, Chemical Potential-9, Molecular Parameters-9, Lennard-Jones-Devonshire-6.

Abstract: The statistical mechanical solid solution theory of van der Waals and Platteeuw has been applied to study the experimentally determined three-phase hydrate-water rich liquid-gas locus of methane-, argon-, and nitrogen-water up to an upper pressure of 50,000 lb./sq.in.abs. The results of calculations show that the solid solution theory can be applied to the hydrates quite well when the caged gas is a spherical molecule but can deviate for slightly nonspherical molecules at high pressure. At higher pressures the hydrate numbers calculated from this theory approached the number of $5\frac{3}{4}$ which is based on complete occupation of all the cavities.

Vapor liquid equilibria for aqueous sulfuric acid, Gmitro, John Irving, and Theodore Vermeulen, **A.I.Ch.E. Journal, 10,** No. 5, p. 740 (September, 1964).

Key Words: A. Sulfuric Acid-8, Water-5, Vapor Pressure-7, Vapor Composition-7, Partial Pressure-7, Azeotrope-7, Temperature-6, Concentration-6, Partial Molar Quantities-10, Dissociation Constant-10, Thermodynamics-10, Computer-10. B. Water-8, Sulfur Trioxide-8, Sulfuric acid-5, Partial Pressure-7, Temperature-6, Concentration-6, Partial Molal quantities-10, Thermodynamics-10.

Abstract: The vapor pressure and the composition of the vapor phase over aqueous solution acid are calculated from liquid-phase partial molal thermodynamic quantities. Partial pressures of water, sulfuric acid gas, and sulfur trioxide are given from —50° to 400°C. between 10 and 100 wt. % acid. Descriptive graphs and tables of partial pressures appear in the paper.

Turbulence energy and intensity spectra in a baffled, stirred vessel, Kim, W. J., and F. S. Manning, **A.I.Ch.E. Journal, 10,** No. 5, p. 747 (September, 1964).

Key Words: Stirred Vessels-1, Methocel-4, Water-5, Impeller Size-6, Revolutions Per Minute-6, Average Velocities-7, Velocity Fluctuations-7, Mixing-8, Intensity Spectra-8, Turbulent Decay-9, Mechanism of Fluid Mixing-9, Transducer Probe-10, Electronic Equipment-10.

Abstract: The radial components of the turbulence energy and intensity spectra within a stirred, baffled tank are measured with a transducer probe. Average velocities are also measured with Pitot and Prandtl tubes. The measured energy spectra show a $k^{-5/3}$-relation in the low wave number range and a $k^{-10/3}$-relation in the higher ranges. This behavior is found regardless of impeller size, motor speed, and position in tank, thus indicating that the decay of energy spectra is self-preserving. Tht shape of the intensity spectra is the same as that of the concentration spectra measured by Manning.

Thickness distribution in a sheet formed by impinging jets, Hasson, David, and Ralph E. Peck, **A.I.Ch.E. Journal, 10,** No. 5, p. 752 (September, 1964).

Key Words: A. Thickness-7, 8, Liquid-9, Sheet/Film-2, 8, Jets-1, Impinging/Colliding-, Fluid Flow-6, Ideal-. B. Jets-1, Impinging/Colliding-, Sheet/Film-2, Streamline Pattern-6, 8.

Abstract: The thickness distribution in a liquid sheet formed by impinging jets is analyzed in accordance with an ideal flow model. The solution obtained is shown to be in excellent agreement with all experimental data reported in the literature.

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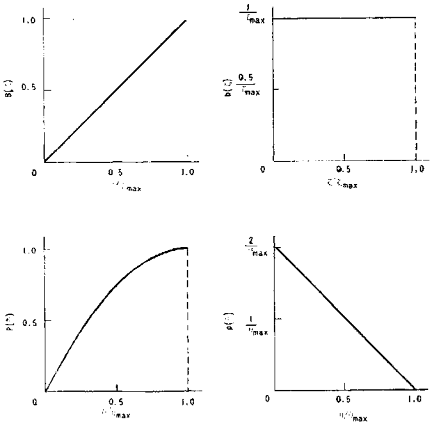

Fig. 4. Distribution and density functions for $b(\zeta) = 1/\zeta_{max}$.

CONCLUSIONS

The size distributions of spherical bubbles, the time-duration distribution of the probe output signal, and the volumetric void fraction have been derived, subject to assumptions stated in the analysis. These results are presented as an approach to interpret property measurements in two-phase fluids. The basic ideas may be applied to local property measurements by replacing area ratios with space-dependent probabilities. This could be a useful by-product of future studies in local measurements. As more work is done on the time and space structure of two-phase flow, these few geometric descriptions should aid in the understanding of local measurements and local fluid properties.

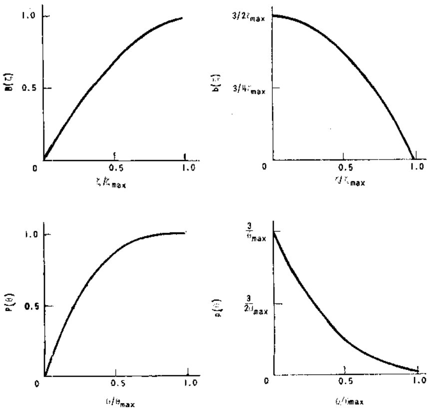

Fig. 5. Distribution and density functions for $b(\zeta) = 3/2\ \zeta_{max} (1 - \zeta^2/\zeta^2_{max})$.

ACKNOWLEDGMENT

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NOTATION

riable of integration

A = projected area of a gas bubble on plane perpendicular to flow

A_p = pipe area perpendicular to flow

$b(\zeta)$ = density function of bubbles striking the probe

$B(\zeta)$ = bubble size distribution function of $b(\zeta)$

C_1 = constant relating chord pierced by probe to bubble diameter

D_B = gas bubble diameter

$h(\zeta)$ = density function of bubbles passing the probe

$H(\zeta)$ = bubble size distribution function of $h(\zeta)$

n = number of equal bubbles in a group striking the probe during measurement time, τ_m

$p(\theta)$ = density function of probe signal duration

$P(\theta)$ = distribution function of probe signal duration

P_1 = probability that a bubble within the pipe strikes the probe

q = rate at which bubbles strike the probe

Q = rate at which bubbles pass the probe

r = spherical bubble radius

R = cylindrical pipe radius

$\mu(\zeta)$ = $U(r)/R$ = normalized bubble velocity

U = bubble velocity

V = bubble volume

U_B = bubble velocity in reference *1*

Z = dummy variable for $\zeta = r/R$

Greek Letters

α = void fraction or volumetric ratio of gas to total volume

δ = chord of a bubble pierced by probe

$\bar{\delta}$ = average chord of equal bubbles pierced by probe

ζ = r/R = bubble radius divided by pipe radius

ζ_{min} = r_{min}/R = minimum bubble size contributing to $p(\theta)$

ζ_{max} = r_{max}/R = maximum bubble size in pipe

θ = time duration of probe output signal

θ_B = time duration of probe output signal in reference *1*

$\bar{\theta}(r)$ = average bubble contact time

ρ = distance from center of bubble to chord

τ_m = time duration of measurement

τ_g = gas contact time during measurement

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